Activation of Directed Neuron Networks Via Controlled Synchronization

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***Resumen*— In this contribution we present the activation of directed ensembles of neurons using controlled synchronization. A network of neurons in a resting state is activated and a prescribed pattern is imposed on the network. Some nodes in the network are connected only in one direction and some others are bidirectional. The idea is to induce or to impose a desired behavior on the network aplling a control action to at least node, such that the network reproduces a regular spiking-bursting behavior. We propose a nonlinear controller to control one neuron in the network and then by means of this controlled neuron and the coupling strength the synchronization of the network is achieved. A numerical simulation is presented in order to illustrate the controlled synchronization of a Scale-Free network of Hindmarsh-Rose neurons.**

Keywords: Neuron Synchronization, Controlled Synchronization.

I. Introduction

A complex network is a collection of dynamical systems connected by a coupling strength. Its study has attracted a lot of attention for many interesting problems and application in real world, see for wide reviews (Albert and Barabasí, 2002), (Wang and Chen, 2003) and (Boccaletti et.al., 2006). Real problems can be seen as a network model for instance in biological systems, electronic circuits, social communities, diseases spreading, neural systems, etc. Complex networks present many challenges, for example the problem of *synchronization of networks with nonidentical nodes*, which means that the systems in nodes are strictly different; *dynamic topology* can be seen as the structural change of the network along the time; *the growth dynamics* concerns on how the network increases or reduces the number of nodes; *synchronization of the network in a chaotic attractor* (Strogatz, 2001). Recently it has been reported that a network with nonidentical nodes can be synchronized in a chaotic attractor, however, a modification of the coupling equation was required in order to obtain the synchronous behavior (Perales et.al., 2009). Other interesting point in synchronization is that the network behavior is a result of the collective dynamics, the coupling strength, the network topology and the complexity in the nodes. In this sense, in terms of neuron networks there is no feasible information about the topology and the interconnection of neurons. Thus, a question arises, *Can be induced a desired behavior into a synchronized complex network?*. An approach can be obtained from controlling some nodes, this is, some nodes are forced to track a prescribed reference such that the synchronous behavior of the network also tracks such a reference.

Dynamical networks can be used to represent real-world biological systems including ensembles of cells in which the most significant phenomenon is the activation of the membrane potential of the neuron (Widmaier et.al., 2007). The activity of neurons is characterized by spiking-bursting behavior of the membrane potential, this behavior is responsible for the transmission and processing of information. It is known that in living organisms this transmission of information can failed due to malfunction of the synapses between some neurons. In such a case the global behavior of the ensemble tends to a resting behavior, even if some neurons in the ensemble behaves in an appropriate way, however, the inhibited neurons inhibit the global behavior. An approach to activate theses neurons is to consider the induction of a proper dynamics provided by an artificial neuron in the ensemble by means of controlling some neurons. In this way it is possible to activate the synchrony and the coordinated operation of the biological network. The artificial neuron can be seen as a synchrony controller that imposes an activation pattern in the ensemble of live neuron that are in inhibit operation (Pinto et.al., 2000).

In this contribution we begin with a simplified model of neurons interconnected forming a network with a given topology and with some of the elements in the network inhibited such that the ensemble behavior is almost inhibited. Then we construct a feedback control system acting on one neuron, thus the controller induces the desired behavior to the controlled neuron, such behavior could be given by an artificial neuron. The general idea is to design a nonlinear controller to stabilize the synchronization error system obtained from the controlled and the reference node. The synchronization error is stabilizes at the origin such that the trajectories of the controlled node tracks the reference trajectories. The controller is obtained from a diffeomorphic transformation and exploiting the controllability and observability for nonlinear systems. The Transformation is such that the synchronization error system is transformed into a fully or partially linearizable system via feedback. Therefore, the control law is obtained via the Lie derivatives of the output functions along the vector fields of the synchronization error system. The controller obtained is applied to one or several nodes in the network in order to activate the ensemble. As was mentioned above the topology and the connections between neurons in a real network is not clear, therefore we consider a scale-free network (Barabási et.al., 2001), the aim of use this kind of network is that they are very close to model real processes, for instances the sexual contacts in a social network is modeled as a scale-free network (Cubukcu et.al., 2003), also the World Wide Web is modeled as a scale-free network (Albert et.al., 1999). Therefore, we assume that the neuron ensemble are represented as a complex scale-free network.

The rest of the manuscript is organized as follows: Section 2 describes the problem of the activation of inhibited neural networks; Section 3 presents the results on the activation controller; in Section 4 simulation results are provided and finally Section 5 closes the contribution with concluding comments.

II. Problem Description

II-A. Model of a Single Neuron

In our analysis each node in the network is a neuron having a description given by the (HR) model:

$$\begin{aligned} \dot{x}_{i1} &= \alpha x_{i2} + \beta x_{i1}^2 - \gamma x_{i1}^3 - \delta x_3 + I_i \\ \dot{x}_{i2} &= \epsilon - \varepsilon x_{i1}^2 - x_{i2} - \zeta x_{i4} \\ \dot{x}_{i3} &= \eta(-x_{i3} + S(x_{i1} + h)) \\ \dot{x}_{i4} &= \theta(-\vartheta x_{i4} + \iota(x_{i2} + \kappa)) \end{aligned} \tag{1}$$

where $x_i = (x_{i1}, x_{i2}, x_{i3}, x_{i4})^\top \in \mathbf{R}^4$ are the state variables of the i-th neuron; with $x_{i,1}$ being the membrane potential, $x_{i,2}$ is the fast current in the ion dynamics, $x_{i,3}$ is the slow current whether $\eta \ll 1$ and $x_{i,4}$ represents an even slower dynamics with $\theta < \eta \ll 1$ to model the calcium exchange between the intracellular stores and the cytoplasm. The model parameters are chosen in such manner that some neurons reproduce spiking-bursting activity, whereas for few neurons the parameter α is chosen in such a way that the corresponding neurons are in an inhibitory state. In Figure 1a) a spiking-bursting activity is shown, this behavior is the desired to be imposed into the entire ensemble, Figure 1b) illustrates the inhibited behavior, we consider that in the ensemble some neurons are in this regime. In order to obtain an inhibitory state of the neuron model we assume the parameter $\alpha < 1$, thus the fast current influence in the neuron is diminished and the spiking-bursting regime is reduced or inhibited and the operation of the neurons become a fixed point as presented in Figure 1b).

II-B. Ensemble Description

We consider that the neurons are coupled via their electrical activity (represented by $I_i + I_i^u$) which from the sequel we consider that it is the unique way to connect or communicate with other neurons in the ensemble and I_i^u is the current entering to the i-th neuron which comes from the neurons connected to the neuron. Therefore, the neuron i-th transmits its current $x_{i,1}$ to the neuron jth via the I_j^u

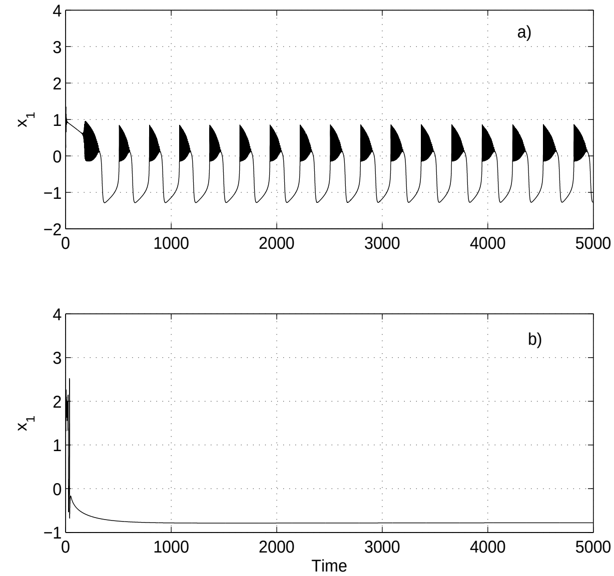

Figura 1. Inhibited neuron using the HR model parameter, it is stabilized at an equilibrium.

current entering. To this end, a network or ensemble with N linearly coupled identical neurons with model given by 1 will have a state description given by

$$\dot{x}_i = F(x_i) + I_i^u \tag{2}$$

for $i = 1, ..., N$ where $I_i^u = c\sum_{j=1}^{N} a_{ij}\Gamma x_j$, x_i are the state variables for the i-th neuron of the ensemble or node of the network; $F : \mathbf{R}^4 \to \mathbf{R}^4$ is the HR smooth vector field representing the dynamics of the ith neuron, $\Gamma = diag[1, 0, 0, 0]$ is the matrix which determines which states in nodes are connected and $\mathcal{A} = \{a_{ij}\}$ is the coupling matrix describing the connectivity of the ensemble. The ensemble considered in this contribution is a scale-free networks, where few nodes are highly connected whereas most of the nodes posses few connections as illustrated in Figure 2.

We have considered two sets of neurons, one for the inhibited and the other for the spiking-bursting neurons, where 21 neurons are active whereas, only 9 neurons are inhibited. The behavior in the network is shown in Figure 3. The neurons were connected at time $t = 2000$sec. and the behavior is clearly synchronized at an equilibrium, in other words, the active neurons instead of activated the resting neurons, they follow the inactive behavior.

Therefore, we look for an answer to the question stated in the Introduction, which can be treated as a control problem. Thus, we propose to design a controller u, such that the entire controlled network (2) be activated via the induction of the desired behavior $x_{Ref,1}$, such a control action is injected in some neurons, such that the resulting dynamics of the network be synchronous with the reference neuron.

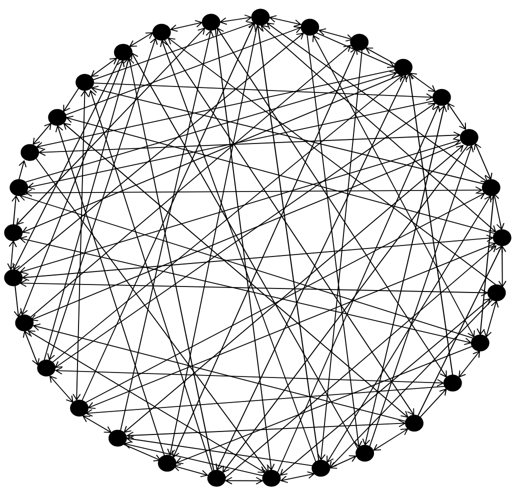

Figura 2. Scale - Free network considered for the ensemble of HR neurons.

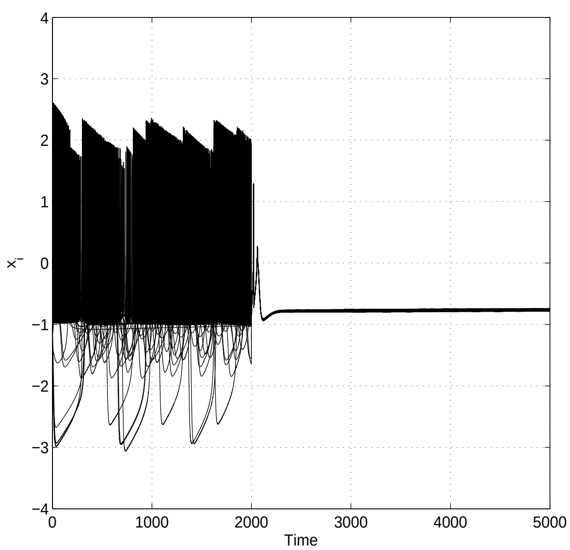

Figura 3. Synchronous behavior of the ensemble in inhibit regime.

III. Imposing an Activation Pattern via Controlled Synchronization

The main idea is to control at least one neuron, therefore, without lost of generality, we consider that the controller is applied to the first $k < N$ neurons in the ensemble, then the controlled network becomes

$$\begin{aligned} \dot{x}_{Ref} &= F(x_{Ref}) \\ \dot{x}_k &= F(x_k) + c\textstyle\sum_{j=1}^{N} a_{kj}\Gamma x_j + Bu_k \\ \dot{x}_i &= F(x_i) + c\textstyle\sum_{j=1}^{N} a_{ij}\Gamma x_j \end{aligned} \quad (3)$$

where x_{Ref} is the state vector of the reference neuron, $k = 1, 2, \cdots, N_c$ is the number of controlled neurons. The reference neuron is not influenced by the rest of the neurons, B is the input control vector to the controlled neuron, a_{kj} are the links between the controlled neurons and the rest of the neurons in the ensemble, and u_k is the control action applied to induce the behavior of the reference neuron. The parameters that can be designed are c and the control gains of the control actions u_k, for simplicity we shall consider that the links between nodes are equal to the unity. Therefore, we have to design the coupling strength and the control parameters in the controller such that the network synchronizes to the reference neuron x_{Ref}.

III-A. *Controller Design*

The induction of a synchronous behavior in the network can be achieved by stabilizing a dynamical system which represents the error dynamics between the reference neuron and the controlled neuron. The synchronous behavior in the network is obtained since the non controlled nodes are linked to the controlled one and by means of the coupling strength. Let us define from eq.(3) the following synchronization error system for the kth controlled neuron and the ensemble neurons, with $\chi_k = x_{Ref} - x_k$

$$\begin{aligned} \dot{\chi}_{k,1} &= \alpha_k \chi_{k,2} + \beta_k \chi_{k,1}^2 - \gamma_k \chi_{k,1}^3 - \delta_k x_{k,3} + \Delta I_k + \\ &\quad \varphi_{k,1}(x_{Ref}, \chi_k, x_i) - u \\ \dot{\chi}_{k,2} &= \Delta\epsilon_k - \Delta\varepsilon_k \chi_{k,1}^2 - \chi_{k,2} - \zeta_k \chi_{k,4} + \varphi_{k,2}(x_{Ref}, \chi_k) \\ \dot{\chi}_{k,3} &= \eta_k(-\chi_{k,3} + S_k(\chi_{k,1} - \mu_k)) + \varphi_{k,3}(x_{Ref}, \chi_k) \\ \dot{\chi}_{k,4} &= \theta_k(-\vartheta_k \chi_{k,4} + l_k(\chi_2 - \kappa_k)) + \varphi_{k,4}(x_{Ref}, \chi_k) \\ y_k &= \chi_{k,1} = x_{Ref,1} - x_{k,1} \\ \dot{x}_i &= F(x_i) + c\left(\textstyle\sum_{j=1}^{N} a_{ij}\Gamma x_j\right) \text{ for } i = k+1, \cdots, N \end{aligned} \quad (4)$$

Note that in this new representation the problem reduces to stabilize the error system at the origin. It is important to stress that in the error system the influence of the non controlled nodes in the network are considered in $\varphi_{k,1}(x_{Ref}, \chi_k, x_i)$, therefore, the control command has to compensate them in order to stabilize the error system. Now to determine a stabilizing controller from eq.(4) we first determine the relative degree (Isidori, 1989), from where we found that

$$\begin{aligned} L_{B_k} h(\chi_{k,1}) &= 1 \\ L_F h(\chi_{k,1}) &= \alpha_k \chi_{k,2} + \beta_k \chi_{k,1}^2 - \gamma_k \chi_{k,1}^3 - \delta_k \chi_{k,3} + \Delta I_k + \\ &\quad \varphi_{k,1}(x_{Ref}, \chi, x_i) \end{aligned} \quad (5)$$

without loss of generality, we propose a diffeomorphic transformation given by $\Phi(\chi) = [\chi_{k,1}, \chi_{k,2}, \chi_{k,3}, \chi_{k,4}]^T$ which is an invertible diffeomorphic transformation since the Jacobian matrix is invertible at the point χ^o of the domain, thus the transformed system is given by

$$\begin{aligned} \dot{z}_{k,1} &= L_F h(\Phi^{-1}(z_k)) + L_{B_k} h(\Phi^{-1}(z_k)) u_k \\ \dot{z}_{k,2} &= \Delta\epsilon_k - \varepsilon_k z_{k,1}^2 - z_{k,2} - \zeta_k z_{k,4} + \varphi_{k,2}(x_o, z_k, x_i) \\ \dot{z}_{k,3} &= \eta_k(-z_{k,3} + S_k(z_{k,1} + \mu_k)) + \varphi_{k,3}(x_o, z_k, x_i) \\ \dot{z}_{k,4} &= \theta_k(-\vartheta_k z_{k,4} + l_k(z_{k,2} + \kappa_k)) + \varphi_{k,4}(x_o, z_k, x_i) \\ y &= z_{k,1} \end{aligned} \quad (6)$$

thus from the transformed system one can propose a stabilizing controller as follows

$$u_k = \frac{1}{L_{B_k} h(\Phi^{-1}(z_k))} \left(-L_F h(\Phi^{-1}(z_k)) + K(z_{k,1} - z^*)\right) \tag{7}$$

where $z^* = 0$ is the stabilizing point, note that the controller requires the function $L_F h(\Phi^{-1}(z_k))$ which represents the dynamics of the system and also contains the deviations of the linked states of the network.

To study the stability of the synchronization error system we obtain the internal dynamics which is given by

$$\begin{aligned} \dot{\chi}_{k,2} &= (\epsilon_{k,M} - \epsilon_{k,S}) + \varepsilon_{k,S}\chi_{k,1}^2 - \chi_{k,2} - \zeta_{k,1}\chi_{k,4} - \\ &\quad \varphi_{k,2}(x_{Ref}, \chi, x_i) \\ \dot{\chi}_{k,3} &= \eta_{k,S}(-\chi_{k,3} + S_{k,S}(\chi_{k,1} + h_{k,S})) + \\ &\quad \varphi_{k,3}(x_{Ref}, \chi, x_i) \\ \dot{\chi}_{k,4} &= \theta_{k,S}(-\vartheta_{k,S}\chi_{k,4} + l_{k,S}(\chi_{k,2} + \kappa_{k,S})) + \\ &\quad \varphi_4(x_{Ref}, \chi, x_i) \end{aligned} \tag{8}$$

where the discrepancy functions $\varphi_{k,i}$ are given by

$$\begin{aligned} \varphi_{k,2} &= -(\varepsilon_{k,M} - \varepsilon_{k,S})x_{1,M}^2 - \varepsilon_{k,S}x_{1,M}\chi_{k,1} - \\ &\quad (\zeta_{k,M} - \zeta_{k,S})x_{4,M} \\ \varphi_{k,3} &= (\eta_{k,M} - \eta_{k,S})(-x_{3,M} + (S_{k,M} - S_{k,S})(x_{1,M} + \\ &\quad (h_{k,M} - h_{k,S}))) \\ \varphi_{k,4} &= (\theta_{k,M} - \theta_{k,S})(-(\vartheta_{k,M} - \vartheta_{k,S})x_{4,M} + \\ &\quad (l_{k,M} - l_{k,S})(x_{2,M} + (\kappa_{k,M} - \kappa_{k,S}))) \end{aligned} \tag{9}$$

on the one hand, we consider that the unique parameters which are considered different are the parameters α and the current I, therefore the discrepancy functions are zero, except $\varphi_{k,2}$ which depends on $\chi_{k,1}$. On the other hand, the controller (7) stabilizes the state $\chi_{k,1} \to 0$ as $t \to 0$, therefore, the zero dynamics is given by

$$\begin{aligned} \dot{\chi}_{k,2} &= -\chi_{k,2} - \zeta_{k,1}\chi_{k,4} \\ \dot{\chi}_{k,3} &= \eta_{k,S}(-\chi_{k,3} + S_{k,S}(\chi_{k,1} + h_{k,S})) \\ \dot{\chi}_{k,4} &= \theta_{k,S}(-\vartheta_{k,S}\chi_{k,4} + l_{k,S}(\chi_{k,2} + \kappa_{k,S})) \end{aligned} \tag{10}$$

it is clear that the zero dynamics is bounded and therefore, the synchronization error system is stabilized at the origin. This implies that each neuron in the ensemble are synchronous with the reference neuron. In this way the controlled nodes tracks the reference signal compensating the inputs given by the non controlled nodes in the network, such that, the controlled nodes impose the desired behavior into the other nodes.

IV. Simulation Results

With the previous statements we consider the network arrangement illustrated in Figure 2 with $N = 30$. Now we consider that only one node is being controlled.

The parameters for the neurons in the nodes are $b = 3$; $c = 1$; $d = 0{,}99$; $I = 0$; $e = 1{,}01$; $f = 5{,}0128$; $g = 0{,}0278$; $m = 0{,}0021$; $s = 3{,}966$; $h = 1{,}605$; $v = 0{,}0009$; $k = 0{,}9573$; $r = 3{,}0$; $l = 1{,}619$; except for the parameter a where we consider several values in order to obtain inhibitory behavior in the nodes. Note that the current I is equal to zero except in the reference neuron.

As was illustrated in Figure 3, the ensemble is synchronous in an inhibited behavior. The idea is to induce

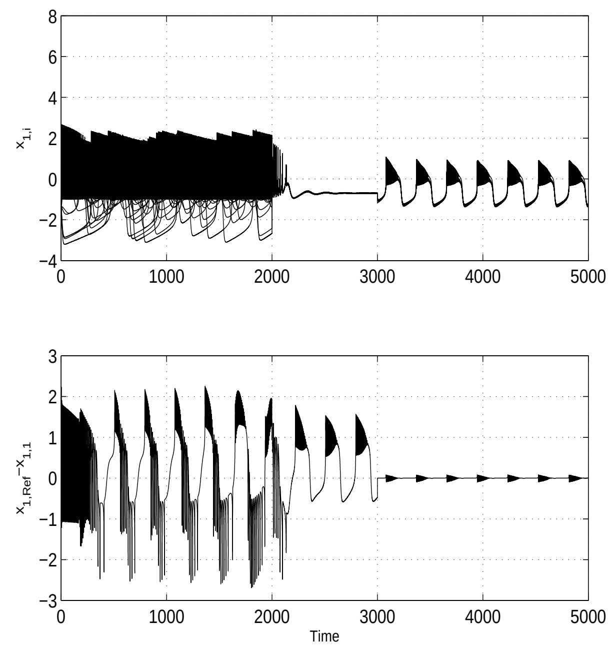

Figura 4. Activation of the inhibit neuron via control of one neuron.

the desired behavior into the ensemble by controlling one neuron. The controller is connected to the controlled neuron for $t \geq 3000$sec. Clearly, the ensemble is reactivated, and the spiking-Bursting behavior is imposed in the whole ensemble. The control gain was $K = 10$ and the coupling strength $c = 10$, the dynamic of the network is illustrated in Figure 4. It is observed that the controller activates the behavior in the network, and for the effect of the coupling between nodes the network synchronizes to the reference neuron, which is the main contribution. There are two aspects that should be mention, the first concerns with the control gain such that the controlled neuron is forced to track the reference signal in such a way that the controlled neuron behavior is induced to the network. The second aspect is related to the coupling and the topology of the network, as one can observe, the controlled neuron is capable to control the ensemble with the coupling strength, this is, as smaller the coupling strength as bigger the control gains of the controller. Therefore, there exists a compromise between the control gains and the coupling strength.

V. Conclusion

In this contribution we present the induction of a desired behavior into a scale-free network of inhibited neurons. The idea was to design a nonlinear controller to induce into some neurons the desired behavior and then by the effect of the interconnection of the network, it is induced the reference behavior to the entire network. We corroborate that an ensemble of inhibited neuron can be reactivated via controlling some nodes and a reference neuron which in practical terms could be an electronic circuit. The result is somewhat conservative since the controller requires information about the systems in the network, therefore an adaptive scheme can be designed to improve the performance. Also we illustrate that a synchronized network can be re-synchronized into a different synchronization manifold which can be established a priori.

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